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Lumped Mass Modelling For the Dynamic Analysis of Aircraft Structures

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ABSTRACT

Aircraft sctructures may be modelled by lumping the masses at particular strategic points and the flexibility or stiffness of the structure is obtained with reference to these points. Equivalent moments of inertia for the section at these positions are determined. The lumped masses are calculated based on the assumption that each point will represent the mass spread on one half of the space on each side. Then these parameters are used in the diferential equation of motion and the eigen characteristics are determined. A comparison will be made with results obtained by other established methods.

The lumped mass approach in the dynamic analysis of complicated structures provides an easier means of predicting the dynamic characteristics of these structures. It involves less computor time and avoids computational errors that are inherent to the numerical solution of complicated systems.

INTRODUCTION

The mass of the hypersonic plane is continuously distributed over the entire structure. Consequently, the real structure has an infinite number of degrees of freedom as far as the dynamic behavior is concerned. However, in the dynamic analysis of structures, it is possible to replace the real structure with an ideal one consisting of a number of lumped masses. These are assumed to be connected to one another through elastic massless elements which, to a certain extant , retain the actual behavior of the original structure. The method of idealizing actual structures bears significantly on the final results in any vibration analysis, and the selection of the method and the number of lumped masses for the system has to made while taking into consideration the various aspects of the structure under study. The skill and experience of the analyst are very helpful in obtaining the best ideal model for the structure.

In idealizing the hypersonic plane, there are certain assumptions which have to be made:

a. Idealization of real strucutres is limited to those structures which deflect in a linearly elastic manner. It is possible to extend the procedure to structures loaded in the plastic region, but the solution of such structures is more complex.

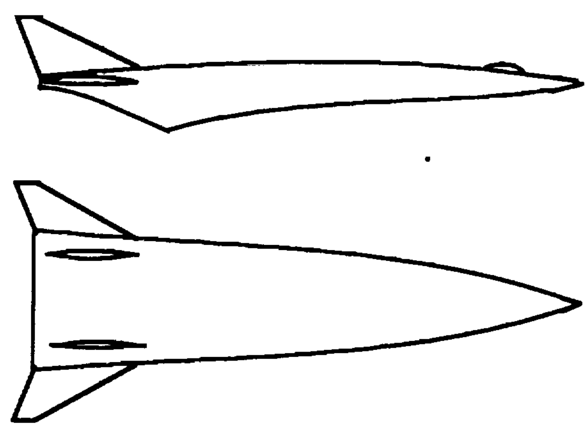
b. To be idealized, a structure must be stable under static loads. This condition applies for both determinate and indeterminate structures.

c. All structures demonstrate a certain amount of damping when they are subjected to dynamic loading conditions. Such damping in structures is controlled by structural hysteresis and by external friction. In the dynamic analysis of ordinary structures damping may be necglected in determining the natural frequencies, but it must be included in the evaluation of mode shapes under resonant conditions.

DEVELOPMENT OF THE METHOD

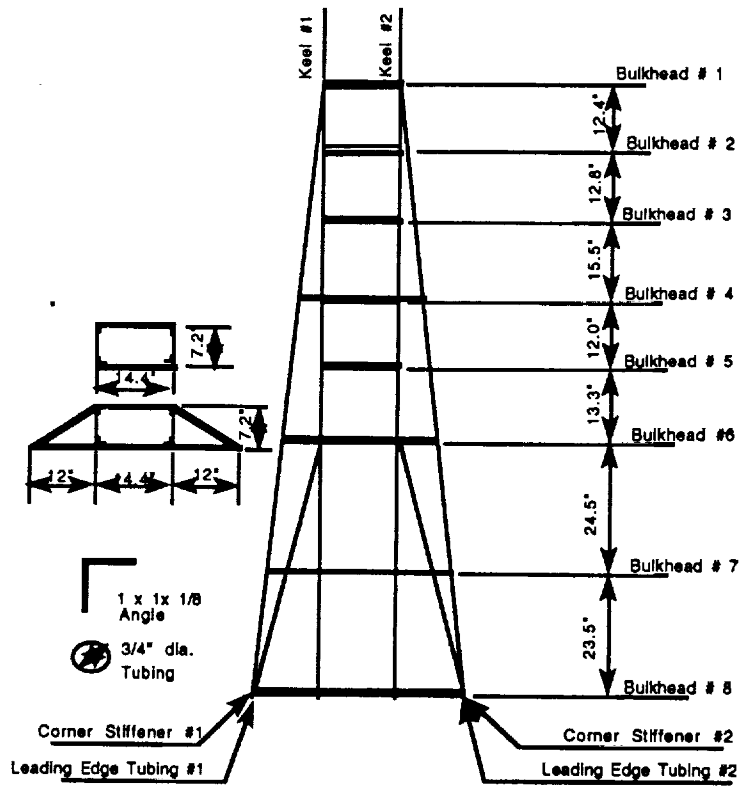
Consider that the frame of the hypersonic plane is fixed at the narrow end to act as a cantilever and that the masses are lumped as seen in Figure 2. In this study the shear and rotary inertia effects are ignored. the dynamic loading on the cantilever beam is the inertia of the moving bodies. The inertia force due to each body is expressed as

$$M_i Z_{im} = - M_i \omega_m^2 Z_{im} \qquad (1)$$



HYPERSONIC PLANE AT MACH 4

(a)



Stiffener Distribution
HYPERSONIC PLANE AT MACH 4

(b)

Figure 1

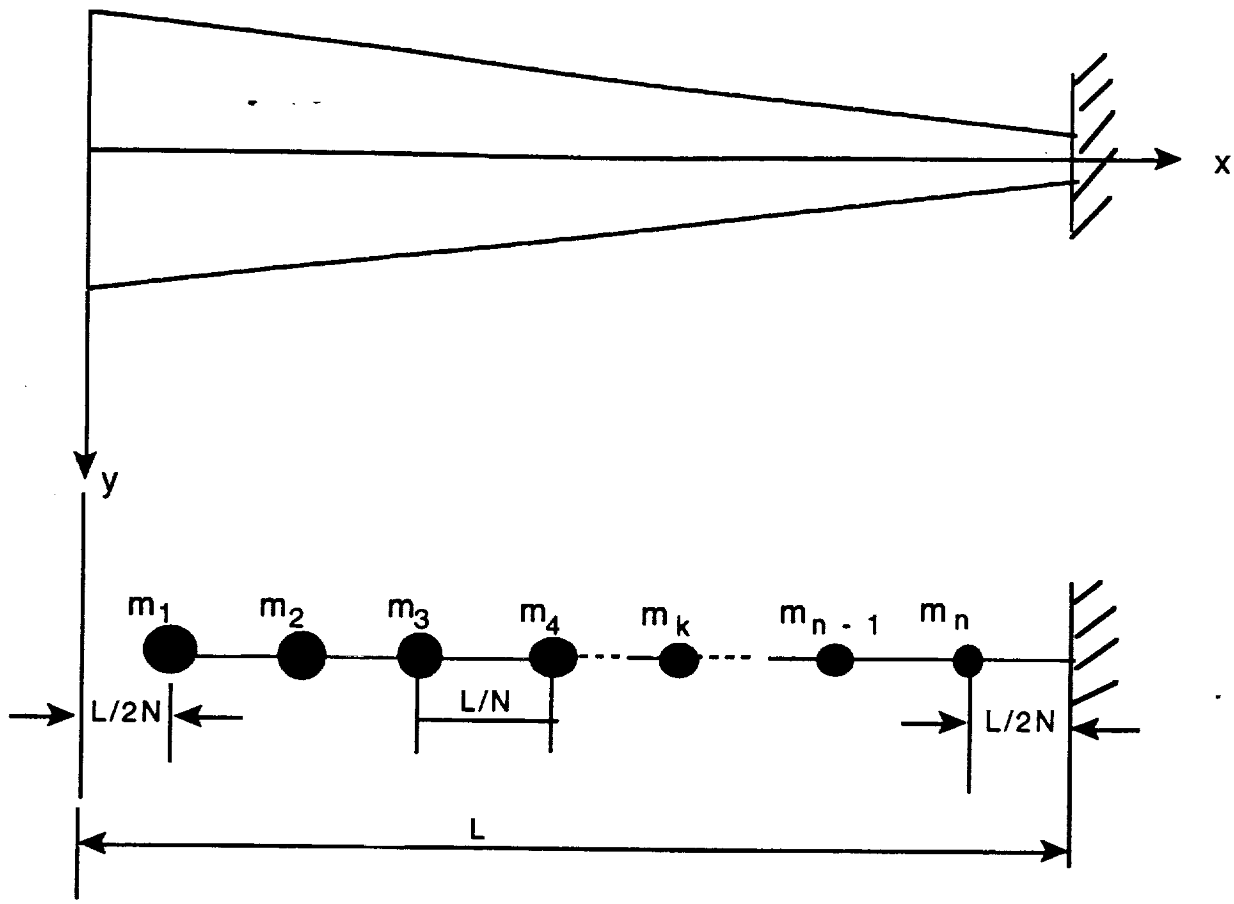


Figure 2

where:

M_i = the lumped mass at a point i

Z_{im} = the deflection of point i in the mth mode

Z_{im} = the acceleration of point i in the mth mode

ω = the circular frequency of the system vibrating in the mth mode

The deflection equation for the structure under dynamic loading due to inertial forces is expressed in the following form:

$$\{Z_m\} = \omega_m^2 \, [A] \, [M] \, \{Z_m\} \qquad (2)$$

where

$\{Z_m\}$ = a column matrix of the displacement of the structure in the mth mode.

$[A]$ = a square matrix of the flexibility coefficients of the structure

$[M]$ = a diagonal matrix of the mass of the structure.

Equation (2) can be expressed in the alternate form as in

$$0 = [D] \, \{Z_m\} \qquad (3)$$

where

$$[D] = [[I] - \omega_m^2 [A][M]] \tag{4}$$

To obtain a non-trivial solution for Equation (3), the determinant of matrix [D] must be identical to zero.

$$0 = |D| \tag{5}$$

The expansion of Equation (4) yields the characteristic equation for the stucture which is a polynomial. The nth degree of the characteristic equation is equal to the rank of the matrix [D]. The roots of this equation represent the eigen values of the structure.

DEVELOPMENT OF THE FLEXIBILITY MATRIX [A]

The model as shown in Figure 2 has displacement in one plane only. The displacements are considered to be lateral and rotational ones. The shear displacements are neglected in this model. The orientation of the model displacements is shown in Figure 3.

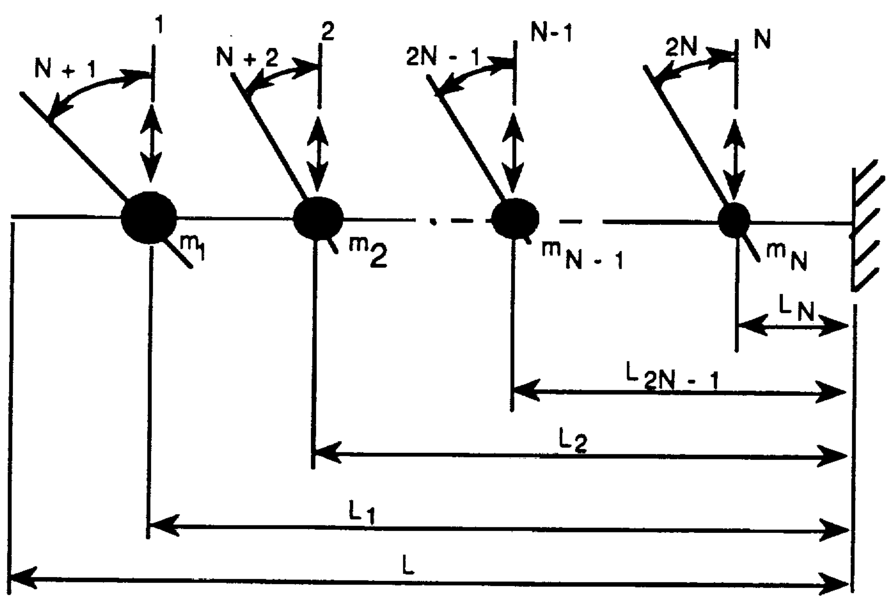


Figure 3

SYMBOLES

N	= Number of lumped masses
L	= Length of beam
I	= Moment of inertia
E	= Young's modulus of elasticity
m_i	= ith mass of the beam/mass moment of inertia

ρ = Density of the beam per unit length

Considering that the beam is divided into equal segments along the longitudinal axis, the length of each segment is given by

$$\Delta = \frac{L}{N} \qquad (6)$$

The mass of a segment of the beam at any point i is

$$m_{ii} = \frac{2.788}{Ng} \left[14.4 + \frac{24}{114} \left(114 - x_i \right) \right], \; i = 1, N \qquad (7a)$$

The function for x_i is given by the following:

$$x_1 = \frac{L}{2N} \qquad (7b)$$

$$x_i = x_{i-1} + \frac{L}{N}, \qquad i = 2, N \qquad (7c)$$

The mass moment of inertia is assumed to be that of a bar with a uniform mass over the length of the segment. It is given by the expression

$$mm_{ii} = m_{ii} \frac{l^2}{12}, \; i = N + 1, 2N \qquad (8)$$

where $l = \frac{L}{N}$, mij = mji = 0 and mmij = mmji = 0

The flexibility matrix [A] in Equation 2 is expressed as follows:

$$[A] = \begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{matrix} \qquad (9)$$

where

[A_{11}] = is NxN matrix that represents the translational diasplacements due to unit lateral forces.

[A_{12}] = is NxN matrix that represents the rotational displacements due to unit lateral forces.

[A_{21}] = is NxN matrix that represents the translational displacements due to unit rotanational forces.

[A_{22}] = is NxN matrix that represents the rotational displacements due to unit rotational forces.

The matrix [A_{11}] is generated from the following equation:

$$a_{ij} = \frac{L^3}{6EI} \times \frac{1}{4N^3} \left[(2N + 1 - 2i)^3 - 3(2N + 1 - 2i)^2 (j-i) + 4 (j-i)^3 \right] \qquad (10)$$

$$a_{ji} = a_{ij} \qquad (11)$$

for i = 1, N and j = i, N.

Matrix [A_{12}] is obtained by taking the derivative of Equation (9).

$$b_{ij} = -\frac{L^2}{2EI} \left(\frac{1}{4N^2} \right) \left[(2N + 1 - 2i)^2 - 4(j - i)^2 \right] \qquad (12)$$

for i = 1, N and j = i , N.

$$b_{ji} = b_{ii}, \quad i = 1, N \text{ and } j = 1, i \tag{13}$$

The development of the matrix [A_{21}] follows from the application of unit moments at the position of the lumped masses on the beam and finding the lateral displacements that ensue from these actions. These displacements are given by

$$C_{ij} = \frac{L^2}{2EI}\left(\frac{1}{4N^2}\right)[(2N + 1 - 2i) - 2(j - i)]^2 \tag{14}$$

for i = 1, N and j = i, N.

$$C_{ji} = C_{ii} + (i - j)\frac{L}{N}\, d_{ii} \tag{15}$$

for i = 1, N and j = i, i - 1

The roatational displacements matrix [A_{22}] due to rotational forces is expressed as

$$d_{ij} = \frac{L}{EI}\left(\frac{1}{2N}\right)[(2N + 1 - 2i) - 2(j-i)] \tag{16}$$

for i = 1, N and j = i, N.

$$d_{ji} = d_{ij}, \quad i = 1, N \text{ and } j = 1, i. \tag{17}$$

Equations (10) through (17) define the flexibilty matrix for the entire structure. Having determined matrices [A] and [M], substitute them in Equation 4 and solve for the eigen values.

EXAMPLE

Consider that the hypersonic plane model is subdivided into three parts. The [A] and [M] matrices are given below.

$$[A] = \begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{matrix}$$

$$[A_{11}] = \frac{L^3}{648EI} \begin{matrix} 125 & 54 & 7 \\ 54 & 27 & 4 \\ 7 & 4 & 1 \end{matrix}$$

$$[A_{12}] = -\frac{L^2}{72EI} \begin{matrix} 25 & 21 & 9 \\ 9 & 9 & 5 \\ 1 & 1 & 1 \end{matrix}$$

$$[A_{21}] = \frac{L^2}{72EI} \begin{matrix} 25 & 9 & 1 \\ 21 & 9 & 1 \\ 9 & 5 & 1 \end{matrix}$$

$$[A_{22}] = \frac{L}{6EI} \begin{matrix} 5 & 3 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 1 \end{matrix}$$

$$[M] = \begin{matrix} M1 & 0 \\ 0 & M2 \end{matrix}$$

where

$$[M_1] = \frac{1}{386.4} \begin{matrix} 33.0 & 0.0 & 0.0 \\ 0.0 & 25.3 & 0.0 \\ 0.0 & 0.0 & 17.7 \end{matrix}$$

and

$$[M_2] = \frac{1083}{386.4} \begin{matrix} 33.0 & 0.0 & 0.0 \\ 0.0 & 25.3 & 0.0 \\ 0.0 & 0.0 & 17.7 \end{matrix}$$

The modulus of elasticity $E = 10^7$ psi. The equivalent average moment of inertia $I = 42.5$ in^4. The length of the beam $L = 114$ in. With the values of the matrices [A] and [M] known, the solution of Equation (5) provides the following frequency results.

The natural frequencies ensuing from the model with three lumped masses are given in the first row for the first six modes. Values in subsequent rows correspond to models of 4, 5, 6, and 7 lumped masses.

Natural Frequency in Hz for a Lumped Mass system

Number of Elements	Number of Modes					
	1	2	3	4	5	6
3	19.54	146.08	396.08	560.60	1329.9	2125.6
4	19.21	138.15	411.18	454.22	1041.8	1492.6
5	19.08	134.70	399.10	509.07	779.19	1661.1
6	19.00	132.87	391.36	558.29	778.32	1247.7
7	18.95	131.78	386.46	603.53	770.65	1273.1

Natural Frequency in Hz for a Finite Element Model

Number of Elements	Number of Modes					
	1	2	3	4	5	6
3	18.11	132.15	385.01	874.00	1656.55	3108.26

CONCLUSION

A lumped mass model for the hypersonic transport airplane has been established. Algorithms for the determination of the flexibility matrix [A] and the mass and mass moment of inertia matrix [M] have been found. The natural frequencies for a 3-lumped mass system have been determined using the lumped mass method and the finite element method. The results from the two methods converge in the lower three modes and diverge in the upper three ones. The lumped mass system requires less computer time than the finite element model. For models with a large number of elements, the lumped mass system is more efficient. Results from both models need to be verified experimentally.